Algebra Standard for Grades Pre-K–2

Instructional programs from prekindergarten through grade 12 should enable all students to—

Expectations

In prekindergarten through grade 2 all students should–

Understand patterns, relations, and functions	• sort, classify, and order objects by size, number, and other properties; • recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another; • analyze how both repeating and growing patterns are generated.
Represent and analyze mathematical situations and structures using algebraic symbols	• illustrate general principles and properties of operations, such as commutativity, using specific numbers; • use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.
Use mathematical models to represent and understand quantitative relationships	• model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.
Analyze change in various contexts	• describe qualitative change, such as a student's growing taller; • describe quantitative change, such as a student's growing two inches in one year.

Algebra

Algebraic concepts can evolve and continue to develop during prekindergarten through grade 2. They will be manifested through work with classification, patterns and relations, operations with whole numbers, explorations of function, and step-by-step processes. Although the concepts discussed in this Standard are algebraic, this does not mean that students in the early grades are going to deal with the symbolism often taught in a traditional high school algebra course.

Even before formal schooling, children develop beginning concepts related to patterns, functions, and algebra. They learn repetitive songs, rhythmic chants, and predictive poems that are based on repeating and growing patterns. The recognition, comparison, and analysis of patterns are important components of a student's intellectual development. When students notice that operations seem to have particular properties, they are beginning to think algebraically. For example, they realize that changing the order in which two numbers are added does not change the result or that adding zero to a number leaves that number unchanged. Students' observations and discussions of how quantities relate to one another lead to initial experiences with function relationships, and their representations of mathematical situations using concrete objects, pictures, and symbols are the beginnings of mathematical modeling. Many of the step-by-step processes that students use form the basis of understanding iteration and recursion.

Patterns are a way for young students to recognize order and to organize their world.

Understand patterns, relations, and functions

Sorting, classifying, and ordering facilitate work with patterns, geometric shapes, and data. Given a package of assorted stickers, children quickly notice many differences among the items. They can sort the stickers into groups having similar traits such as color, size, or design and order them from smallest to largest. Caregivers and teachers should elicit from children the criteria they are using as they sort and group objects. Patterns are a way for young students to recognize order and to organize their world and are important in all aspects of mathematics at this level. Preschoolers recognize patterns in their environment and, through experiences in school, should become more skilled in noticing patterns in arrangements of objects, shapes, and numbers and in using patterns to predict what comes next in an arrangement. Students know, for example, that "first comes breakfast, then school," and "Monday we go to art, Tuesday we go to music." Students who see the digits "0, 1, 2, 3, 4, 5, 6, 7, 8, 9" repeated over and over will see a pattern that helps them learn to count to 100—a formidable task for students who do not recognize the pattern.

E-example 4.1

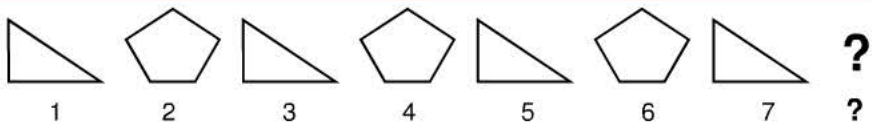
Creating and Predicting Visual Patterns

Teachers should help students develop the ability to form generalizations by asking such questions as "How could you describe this pattern?" or "How can it be repeated or extended?" or "How are these patterns alike?" For example, students should recognize that the color pattern "blue, blue, red, blue, blue, red" is the same in form as "clap, clap, step, clap, clap, step." This recognition lays the foundation for the idea that two very different situations can have the same mathematical features and thus are the same in some important ways. Knowing that each pattern above could be described as having the form AABAAB is for students an early introduction to the power of algebra.

By encouraging students to explore and model relationships using language and notation that is meaningful for them, teachers can help students see different relationships and make conjectures and generalizations from their experiences with numbers. Teachers can, for instance, deepen students' understanding of numbers by asking them to model the same quantity in many ways—for example, eighteen is nine groups of two, 1 ten and 8 ones, three groups of six, or six groups of three. Pairing counting numbers with a repeating pattern of objects can create a function (see fig. 4.7) that teachers can explore with students: What is the second shape? To continue the pattern, what shape comes next? What number comes next when you are counting? What do you notice about the numbers that are beneath the triangles? What shape would 14 be?

Fig. **4.7.**

Pairing counting numbers with a repeating pattern



Students should learn to solve problems by identifying specific processes. For example, when students are skip-counting three, six, nine, twelve, …, one way to obtain the next term is to add three to the previous number. Students can use a similar process to compute how much to pay for seven balloons if one balloon costs 20¢. If they recognize the sequence 20, 40, 60, … and continue to add 20, they can find the cost for seven balloons. Alternatively, students can realize that the total amount to be paid is determined by the number of balloons bought and find a way to compute the total directly. Teachers in grades 1 and 2 should provide experiences for students to learn to use charts and tables for recording and organizing information in varying formats (see figs. 4.8 and 4.9). They also should discuss the different notations for showing amounts of money. (One balloon costs 20¢, or $0.20, and seven balloons cost $1.40.)

Fig. **4.8.**

A vertical chart for recording and organizing information

Cost of Balloons

Number of Balloons	Cost of Balloons in Cents
1	20
2	40
3	60
4	80
5	?
6	?
7	?

Skip-counting by different numbers can create a variety of patterns on a hundred chart that students can easily recognize and describe (see fig. 4.10). Teachers can simultaneously use hundred charts to help students learn about number patterns and to assess students' understanding of counting patterns. By asking questions such as "If you count by tens beginning at 36, what number would you color next?" and "If you continued counting by tens, would you color 87?" teachers can observe whether students understand the correspondence between the visual pattern formed by the shaded numbers and the counting pattern. Using a calculator and a hundred chart enables the students to see the same pattern in two different formats.

Fig. **4.9.**

A horizontal chart for recording and organizing information

Cost of Balloons

Number of balloons	1	2	3	4	5	6	7
Cost of balloons in cents	20	40	60	80	?	?	?

1	2	**3**	4	5	**6**	7	8	**9**	10
11	**12**	13	14	**15**	16	17	**18**	19	20
21	22	23	**24**	25	26	**27**	28	29	**30**
31	32	**33**	34	35	**36**	37	38	**39**	40
41	**42**	43	44	**45**	46	47	**48**	49	50
51	52	53	**54**	55	56	**57**	58	59	**60**
61	62	**63**	64	65	**66**	67	68	**69**	70
71	**72**	73	74	**75**	76	77	**78**	79	80
81	82	83	**84**	85	86	**87**	88	89	**90**
91	92	**93**	94	95	**96**	97	98	**99**	100

Counting by threes

1	2	3	4	5	**6**	7	8	9	10
11	**12**	13	14	15	16	17	**18**	19	20
21	22	23	**24**	25	26	27	28	29	**30**
31	32	33	34	35	**36**	37	38	39	40
41	**42**	43	44	45	46	47	**48**	49	50
51	52	53	**54**	55	56	57	58	59	**60**
61	62	63	64	65	**66**	67	68	69	70
71	**72**	73	74	75	76	77	**78**	79	80
81	82	83	**84**	85	86	87	88	89	**90**
91	92	93	94	95	**96**	97	98	99	100

Counting by sixes

Fig. **4.10.**

Skip-counting on a hundred chart

Represent and analyze mathematical situations and structures using algebraic symbols

Two central themes of algebraic thinking are appropriate for young students. The first involves making generalizations and using symbols to represent mathematical ideas, and the second is representing and solving problems (Carpenter and Levi 1999). For example, adding pairs of numbers in different orders such as 3 + 5 and 5 + 3 can lead students to infer that when two numbers are added, the order does not matter. As students generalize from observations about number and operations, they are forming the basis of algebraic thinking.

Similarly, when students decompose numbers in order to compute, they often use the associative property for the computation. For instance, they may compute 8 + 5, saying, "8 + 2 is 10, and 3 more is 13." Students often discover and make generalizations about other properties. Although it is not necessary to introduce vocabulary such as *commutativity* or *associativity*, teachers must be aware of the algebraic properties used by students at this age. They should build students' understanding of the importance of their observations about mathematical situations and challenge them to investigate whether specific observations and conjectures hold for all cases.

Teachers should take advantage of their observations of students, as illustrated in this story drawn from an experience in a kindergarten class.

> The teacher had prepared two groups of cards for her students. In the first group, the number on the front and back of each card differed by 1. In the second group, these numbers differed by 2.
>
> The teacher showed the students a card with 12 written on it and explained, "On the back of this card, I've written another number." She turned the card over to show the number 13. Then she showed the students a second card with 15 on the front and 16 on the back.

As she continued showing the students the cards, each time she asked the students, “What do you think will be on the back?” Soon the students figured out that she was adding 1 to the number on the front to get the number on the back of the card.

Then the teacher brought out a second set of cards. These were also numbered front and back, but the numbers differed by 2, for example, 33 and 35, 46 and 48, 22 and 24. Again, the teacher showed the students a sample card and continued with other cards, encouraging them to predict what number was on the back of each card. Soon the students figured out that the numbers on the backs of the cards were 2 more than the numbers on the fronts.

When the set of cards was exhausted, the students wanted to play again. “But,” said the teacher, “we can’t do that until I make another set of cards.” One student spoke up, “You don’t have to do that, we can just flip the cards over. The cards will all be minus 2.”

As a follow-up to the discussion, this teacher could have described what was on each group of cards in a more algebraic manner. The numbers on the backs of the cards in the first group could be named as “front number plus 1” and the second as “front number plus 2.” Following the student’s suggestion, if the cards in the second group were flipped over, the numbers on the backs could then be described as “front number minus 2.” Such activities, together with the discussions and analysis that follow them, build a foundation for understanding the inverse relationship.

Through classroom discussions of different representations during the pre-K–2 years, students should develop an increased ability to use symbols as a means of recording their thinking. In the earliest years, teachers may provide scaffolding for students by writing for them until they have the ability to record their ideas. Original representations remain important throughout the students’ mathematical study and should be encouraged. Symbolic representation and manipulation should be embedded in instructional experiences as another vehicle for understanding and making sense of mathematics.

Equality is an important algebraic concept that students must encounter and begin to understand in the lower grades. A common explanation of the equals sign given by students is that “the answer is coming,” but they need to recognize that the equals sign indicates a relationship—that the quantities on each side are equivalent, for example, 10 = 4 + 6 or 4 + 6 = 5 + 5. In the later years of this grade band, teachers should provide opportunities for students to make connections from symbolic notation to the representation of the equation. For example, if a student records the addition of four 7s as shown on the left in figure 4.11, the teacher could show a series of additions correctly, as shown on the right, and use a balance and cubes to demonstrate the equalities.

Fig. **4.11.**

A student’s representation of adding four 7s (left) and a teacher’s correct representation of the same addition

$$7+7=14+7=21+7=28$$

$$7+7=14$$
$$14+7=21$$
$$21+7=28$$

Use mathematical models to represent and understand quantitative relationships

Students should learn to make models to represent and solve problems. For example, a teacher may pose the following problem:

> There are six chairs and stools. The chairs have four legs and the stools have three legs. Altogether there are twenty legs. How many chairs and how many stools are there?

One student may represent the situation by drawing six circles and then putting tallies inside to represent the number of legs. Another student may represent the situation by using symbols, making a first guess that the number of stools and chairs is the same and adding $3 + 3 + 3 + 4 + 4 + 4$. Realizing that the sum is too large, the student might adjust the number of chairs and stools so that the sum of their legs is 20.

Analyze change in various contexts

Change is an important idea that students encounter early on. When students measure something over time, they can describe change both qualitatively (e.g., "Today is colder than yesterday") and quantitatively (e.g., "I am two inches taller than I was a year ago"). Some changes are predictable. For instance, students grow taller, not shorter, as they get older. The understanding that most things change over time, that many such changes can be described mathematically, and that many changes are predictable helps lay a foundation for applying mathematics to other fields and for understanding the world.